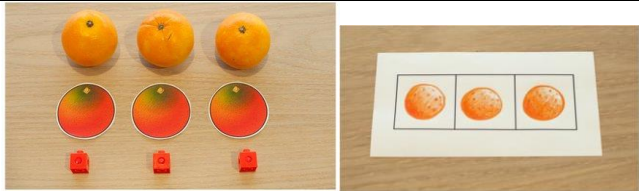
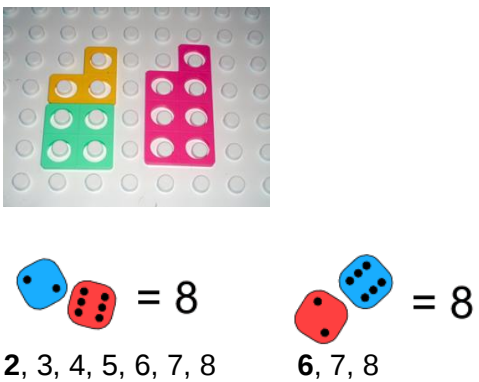
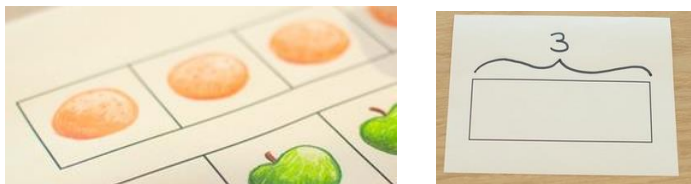
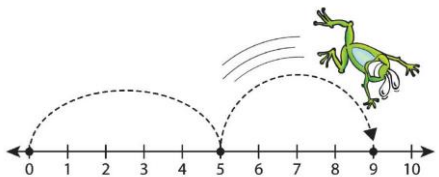
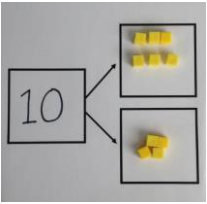
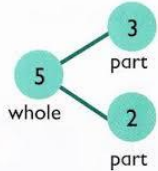
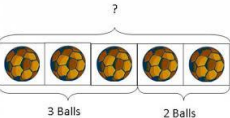
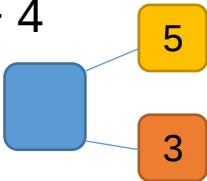
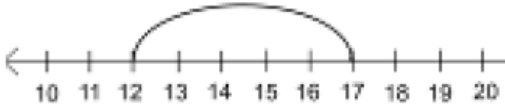
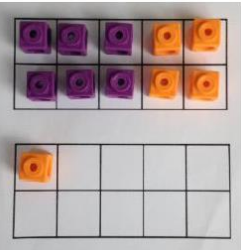
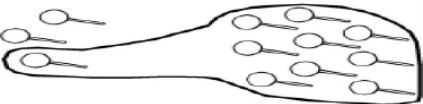
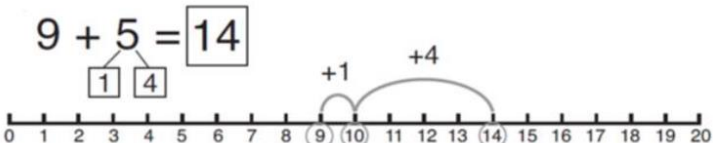
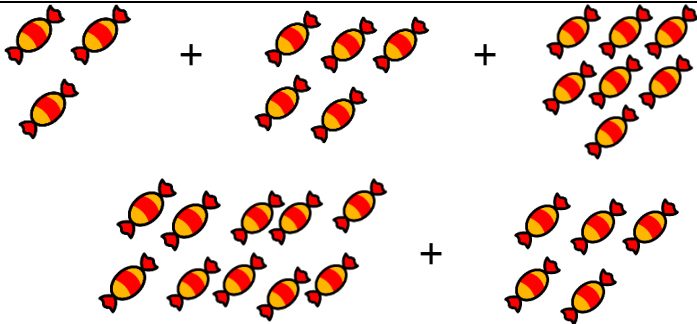
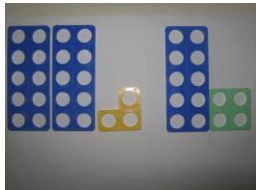
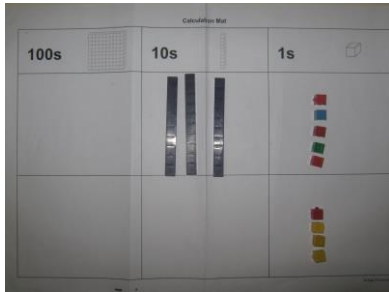
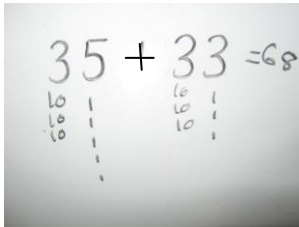


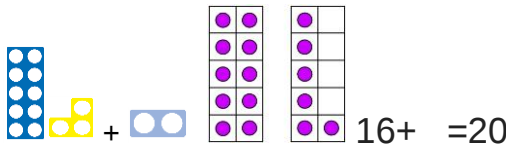
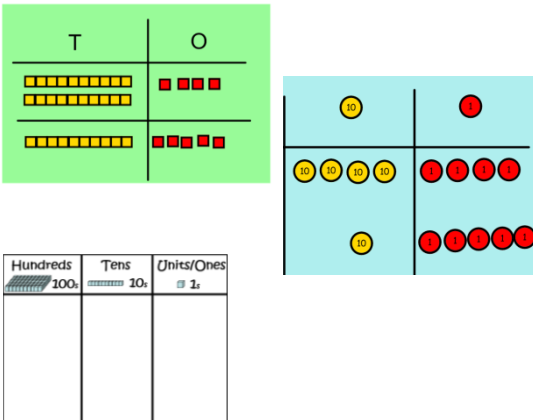
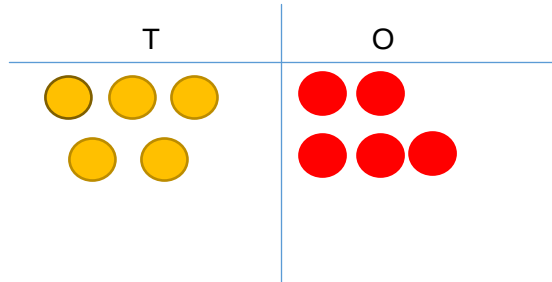
Progression in Calculations

Addition

Objective and Strategies	Concrete	Pictorial	Abstract
EYFS Count reliably with numbers from 1 to 20 - Place them in order and say which number is one more	 Counting a range of familiar objects ensuring development of 1:1 correspondence.	 Start by counting familiar things using blocks or cut-out pictures they can physically line up in a row. For instance counting pieces of fruit, or people in the room. With one block or cut-out picture for each orange or person. Using this add 1 more.	$5 + 1 = 6$ $6 + 1 = 7$ Use pictures of objects to add 1 more - finding how many altogether to use alongside the abstract.
EYFS Using quantities and objects, add two single-digit numbers and count on to find the answer.	 $2 + 6 = 8$ $6 + 2 = 8$ 2, 3, 4, 5, 6, 7, 8 6, 7, 8	 Add objects to the row to understand adding by counting on.  Counting on using a number track or number line.	$6 + 3 = 9$ $7 + 4 = 11$

Year 1 Combining two parts to make a whole: part-whole model	  Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	$4 + 3 = 7$ $10 = 6 + 4$ $16 + 4 = 20$  Use the part-part whole diagram as shown above to move into the abstract.
Year 1 Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	$12 + 5 = 17$  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	$5 + 12 = 17$ Place the larger number in your head and count on the smaller number to find your answer
Year 1 Regrouping to make 10.	  $6 + 5 = 11$ Start with the bigger number and use the smaller number to make 10.	 $3 + 9 =$ Use pictures or a number line. Regroup or partition the smaller number to make 10. $9 + 5 = 14$ 	$7 + 4 = 11$ If I am at seven, how many more do I need to make 10? How many more do I add on now?

Year 2 Adding three single digits	$4 + 7 + 6 = 17$ Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	$\begin{array}{r} 4 + 7 + 6 = 10 + 7 \\ \quad \quad \quad 10 \quad \quad \quad \\ = 17 \end{array}$ Combine the two numbers that make 10 and then add on the remainder.								
Year 2 Addition of 2 digit and 1digit Addition of 2 digit and 10s Addition of 2 two digit numbers	$13 + 6 = 19$  $22 + 10 = 32$  $23 + 14 = 37$ 	Using a calculations mat to ensure equipment is organised and placed in correct column.  $35 + 4 = 39$	Children's recording Identifying the tens and ones within the calculation. 								
Year 2 Commutative law Addition of 2 numbers can be done in any	Cuisenaire $7 + 3 = 10$  $3 + 7 = 10$ Numicon 	After practically using cuisenaire children can draw the Bar Model to demonstrate understanding. <table><tr><td colspan="2">?</td></tr><tr><td>7</td><td>3</td></tr></table>and<table><tr><td colspan="2">?</td></tr><tr><td>3</td><td>7</td></tr></table>	?		7	3	?		3	7	$17 + 4 = 21$ $4 + 17 = 21$ Children's recording noting there is no need to calculate the second calculation due to Commutative Law.
?											
7	3										
?											
3	7										

order															
Year 2 Addition facts to 20 Use related facts to 100	Numicon - addition facts to 20  $16 + _ = 20$ Tens frames and double sided counters	Draw pictures or use of a number line or similar.	Children's Recordings drawing upon related facts $6 + 4 = 10$ $60 + 40 = 100$ $16 + 4 = 20$												
Year 2/3 Column method- no regrouping Year 3 - 3 digit numbers	$24 + 15 =$ Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  <table data-bbox="407 952 631 1123"><tr><th>Hundreds</th><th>Tens</th><th>Units/Ones</th></tr><tr><td>100</td><td>10</td><td>1</td></tr><tr><td></td><td></td><td></td></tr><tr><td></td><td></td><td></td></tr></table>	Hundreds	Tens	Units/Ones	100	10	1							After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	<u>Calculations</u> $21 + 42 =$ $\begin{array}{r} 21 \\ + 42 \\ \hline \end{array}$
Hundreds	Tens	Units/Ones													
100	10	1													

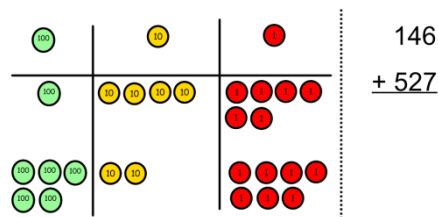
Column method-
regrouping
Year 3 - 3 digit
numbers

Column method-
regrouping
Year 4 - up to 4
digit numbers

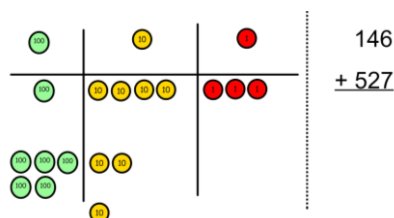
Year 5
Add whole
numbers with
more than 4
digits and
decimals up to 3
places -

Year 6
Solve addition
and subtraction
multi step
problems in
context
deciding which
operations and

Make both numbers on a place value
grid.



Add up the units and exchange 10
ones for one 10.

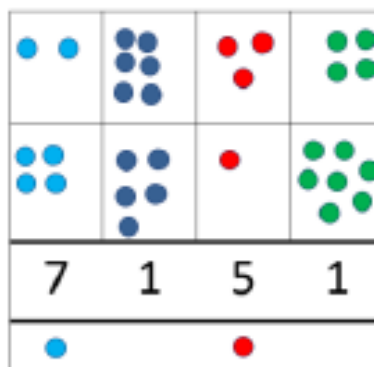


Add up the rest of the columns,
exchanging the 10 counters from one
column for the next place value
column until every column has been
added.

This can also be done with Base 10 to
help children clearly see that 10 ones
equal 1 ten and 10 tens equal 100.

As children move on to decimals,
money and decimal place value
counters can be used to support
learning.

Children can draw a pictorial representation of the
columns and place value counters to further support
their learning and understanding.



Partially numbered and blank numberlines.

Year 3
Start by partitioning the
numbers before moving on
to clearly show the
exchange below the
addition.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array} \quad \text{*****}$$

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

As the
children move
on, introduce
decimals with
the same number of
decimal places (year 4
money) and different.
Money can be used here.

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array}$$

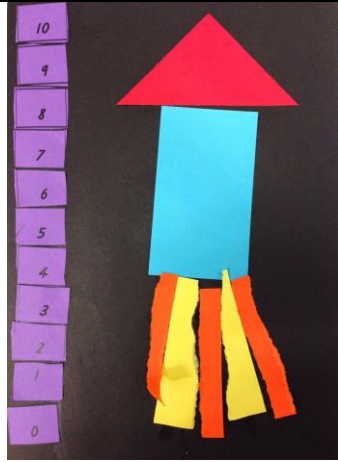
$$\begin{array}{r} £ 2 3 . 5 9 \\ + £ 7 . 5 5 \\ \hline £ 3 1 . 1 4 \\ 1 1 1 \end{array}$$

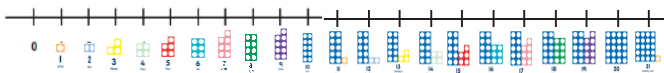
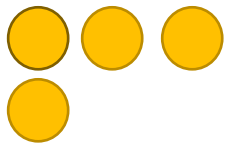
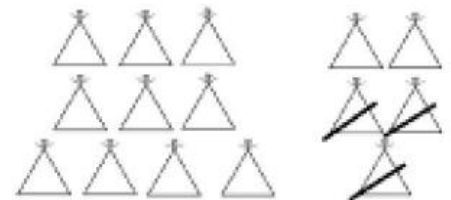
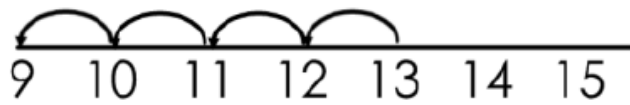
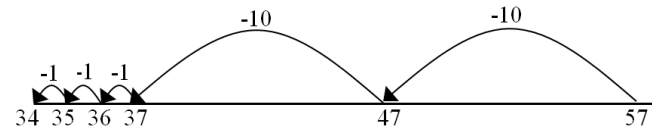
$$\begin{array}{r} 2 3 . 3 6 1 \\ 9 . 0 8 0 \\ 5 9 . 7 7 0 \\ + 1 . 3 0 0 \\ \hline 9 3 . 5 1 1 \\ 2 1 2 \end{array}$$

methods to use
and why.

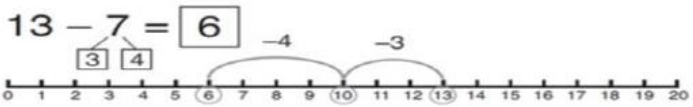
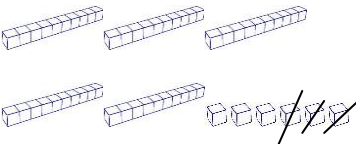
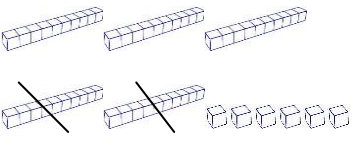
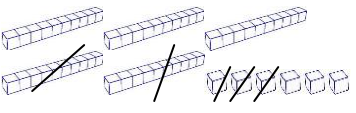
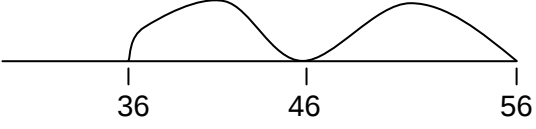


Subtraction

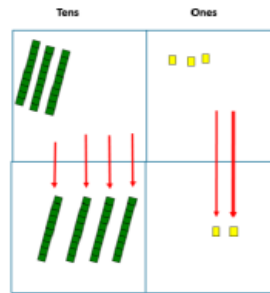
Objective and Strategies	Concrete	Pictorial	Abstract
EYFS Count reliably with numbers from 1 to 20, place them in order and say which number is one less than a given number.	 How many teddies would there be if we had 1 less?	 There are 9 children in a group. One child leaves. How many are in the group now?	$5 - 1 = 4$ $7 - 1 = 6$ Use pictures of objects to find 1 less - finding how many are left to use alongside the abstract.
EYFS Using quantities and objects subtract two single-digit numbers and count back to find the answer.	 Can you take away 2 jewels? How many are left?	 Can you find 7? Now take away 3. What number have you landed on?	$6 - 3 = 3$ $9 - 4 = 5$

			
Year 1 Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  $6 - 2 = 4$  	Cross out drawn objects to show what has been taken away.  $15 - 3 = 12$	$18 - 3 = 15$ $13 - 8 = 5$
Year 1 Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. $13 - 4$  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  Year 1/2 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Year 1 Find the difference	Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference Year 1	Count on to find the difference. Comparison Bar Models Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them. Draw bars to find the difference between 2 numbers. Year 2	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Year 1 Part Part Whole Model	Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? $10 - 6 =$	Use a pictorial representation of objects to show the part part whole model.	Move to using numbers within the part whole model. Up to 20

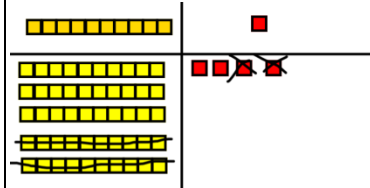
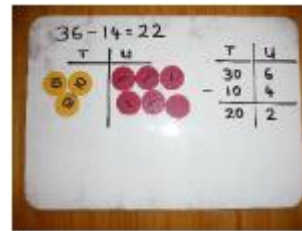
Year 1 Make 10	14 – 5 = Year 1  Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer. Year 2	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?
Year 2 Sub 2 digit take away ones 2 digit take away 10s 2 digit take away 2 digit Subtraction not in any order	56 – 3 = 53  56 – 20 = 36  56 – 23 = 33  Explore subtraction in any order - can we calculate 23 – 56 ? Compare to addition in any order.	Using a calculations mat to ensure equipment is organised and placed in correct column. Drawing blank number line. 	Children's recording Identifying the tens and ones within the calculation. Crossing off the appropriate number.

Column method
without
regrouping
Year 3



Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.



Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Draw the Base 10 or place value counters alongside the written calculation to help to show working.

$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

This will lead to a clear written column subtraction.

$$\begin{array}{r} 32 \\ - 12 \\ \hline 20 \end{array}$$

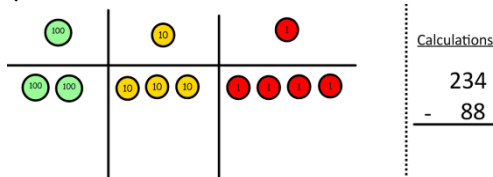
Column method
with regrouping
Year 3 - 3 digit numbers

Year 4 - 4 digit numbers

Year 5 - more than 4 digit numbers

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

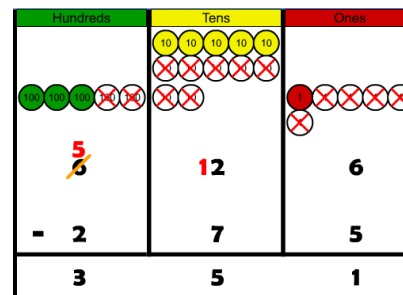
Make the larger number with the place value counters



Calculations

$$\begin{array}{r} 234 \\ - 88 \\ \hline \end{array}$$

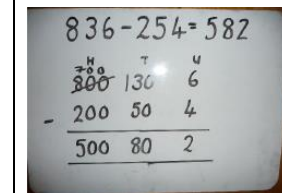
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



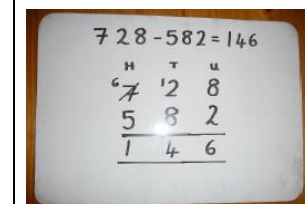
Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



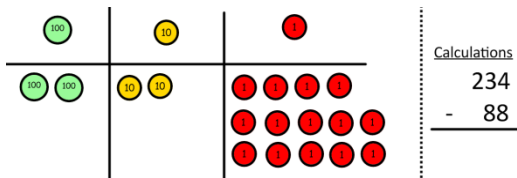
When confident, children can find their own way to record the exchange/regrouping.



Children can start their formal written method by partitioning the number into clear place value columns.

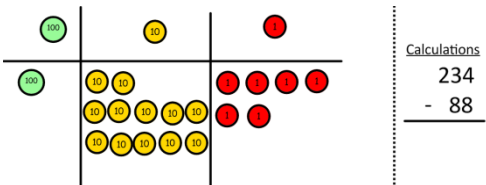
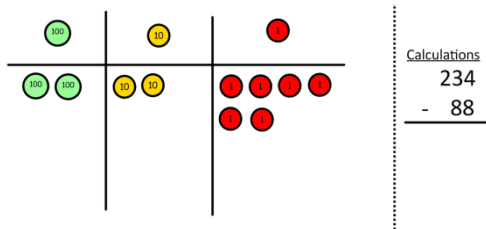


Year 6 - multi step problems

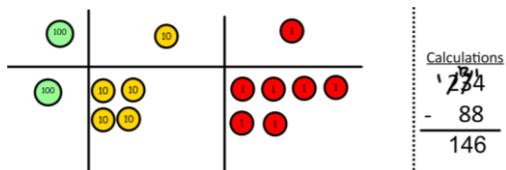


Now I can subtract my ones.

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Now I can take away eight tens and complete my subtraction



Show children how the concrete

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

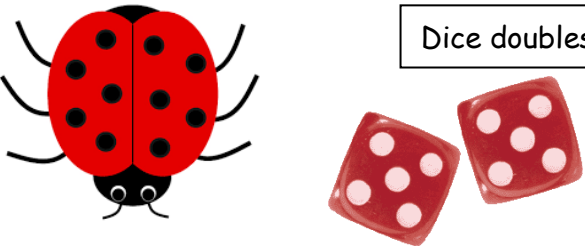
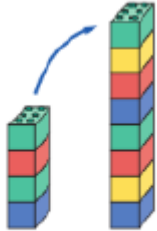
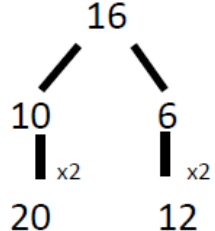
Moving forward the children use a more compact method.

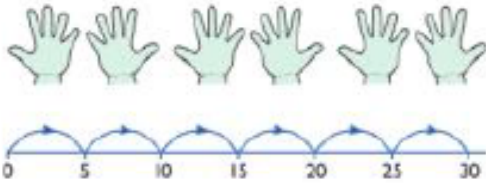
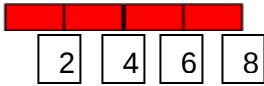
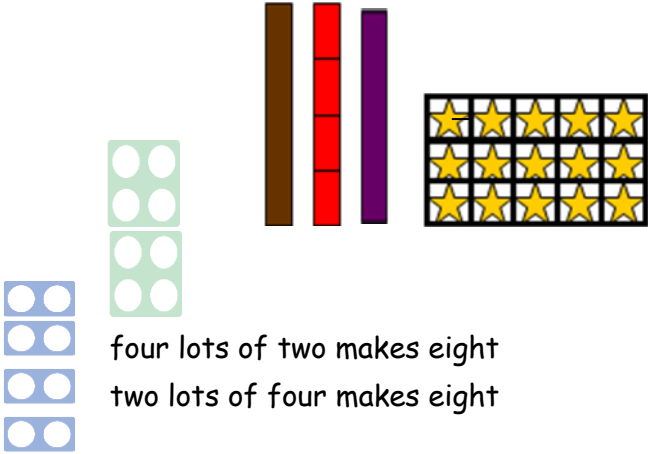
This will lead to an understanding of subtracting any number including decimals.

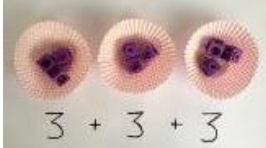
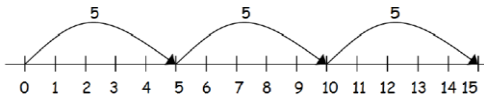
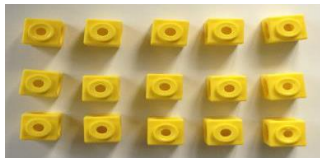
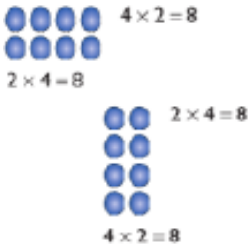
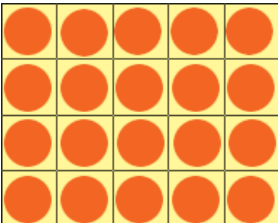
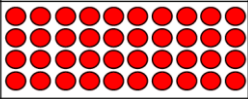
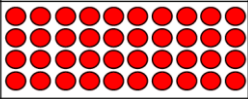
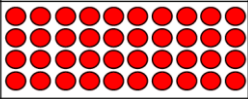
$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad \color{red}{0} \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$

	method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.		
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Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
EYFS Solve problems including doubling	Finding doubles $\square + \square = \square$ $\square + \square = \square$ $\square + \square = \square$ $\square + \square = \square$ $\square + \square = \square$ Find 2 pieces of the same numicon. $\begin{array}{ c } \hline \bullet \bullet \\ \hline \bullet \bullet \\ \hline \end{array}$ $\begin{array}{ c } \hline \bullet \bullet \\ \hline \bullet \bullet \\ \hline \end{array}$ 4 4 8	 Dice doubles Draw dots onto the ladybird to show the same amount on each side. How many in total?	$5 + 5 = 10$ $3 + 3 = 6$
Year 1 Doubling	Use practical activities to show how to double a number.  double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double a number. Double 4 is 8 	 Partition a number and then double each part before recombining it back together. Year 2

Counting in multiples Year 1 2s 5s 10s Year 2 2s 3s 5s 10s	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30 Year 2
Year 1 Arrays with support	 Counting Make connections between arrays, number patterns and counting in twos, fives and tens.  EVERYDAY ARRAYS Explore real life arrays	Using concrete objects, pictorial representations and arrays with the support of an adult - take photographs/draw pictures  Cuisenaire  four lots of two makes eight two lots of four makes eight	Write number sentences (with support) to show what the array represents $2+2+2+2+2=10$ $5+5=10$ $5 \times 2=10$ $2 \times 5=10$

Year 1/2 Repeated addition	 Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10						
Year 2 Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences. 	Draw arrays in different rotations to find commutative multiplication sentences.  4 × 2 = 8 2 × 4 = 8  Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 × 3 = 15 3 × 5 = 15						
Year 3 Grid Method Multiplication and division	Show the link with arrays to first introduce the grid method. <table><tr><td>x</td><td>10</td><td>3</td></tr><tr><td>4</td><td></td><td></td></tr></table> 4 rows of 10 4 rows of 3 Move on to using Base 10 to move	x	10	3	4			Children can represent the work they have done with place value counters in a way that they understand.	Start with multiplying by one digit numbers and showing the clear addition alongside the grid.
x	10	3							
4									

facts 3s 4s 8s

2 digit x 1 digit

towards a more compact method.

x	T	U

4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.

100	10	1

Calculations
4 x 126

Fill each row with 126.

100	10	1
100	10	1
100	10	1
100	10	1
100	10	1

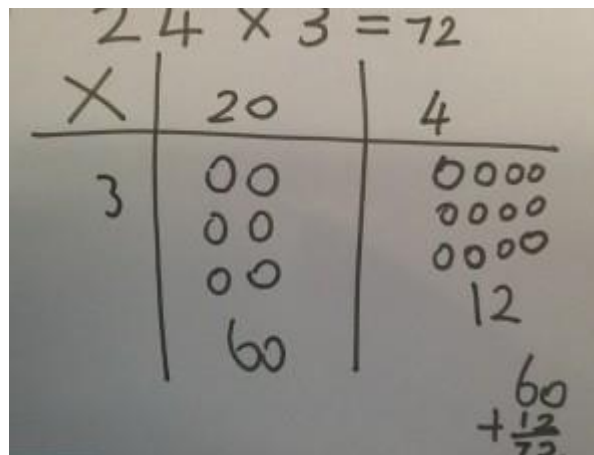
Calculations
4 x 126

Add up each column, starting with the ones making any exchanges needed.

100	10	1
100	10	1
100	10	1
100	10	1
100	10	1

Then you have your answer.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



x	30	5
7	210	35

$$210 + 35 = 245$$

Moving forward Year 4, multiply by a 2 digit number showing the different rows within the grid method.

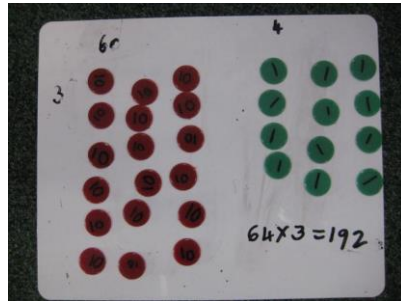
	10	8
10	100	80
3	30	24

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

Year 4

Column
multiplication

Children can continue to be supported by place value counters at the stage of multiplication.



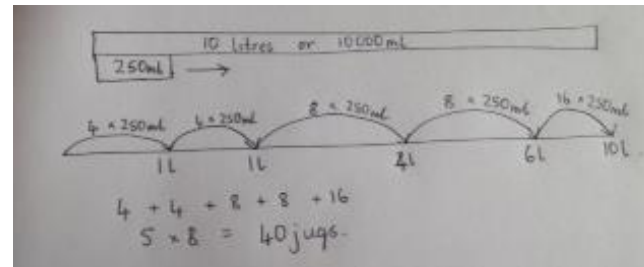
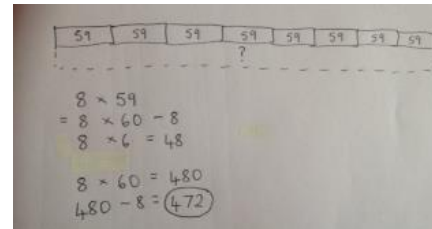
It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Grid alongside the formal 2 digit by 1 digit -year 3/4

Year 5/6

Multiplication -
compact method

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.



Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

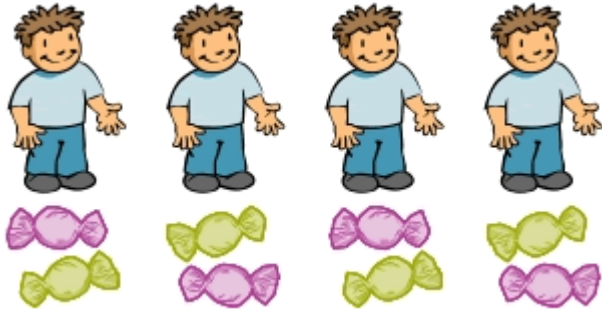
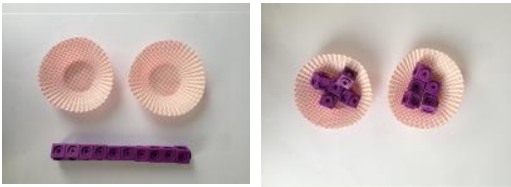
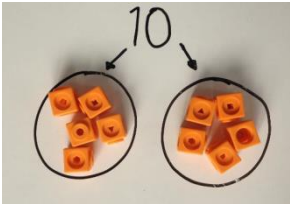
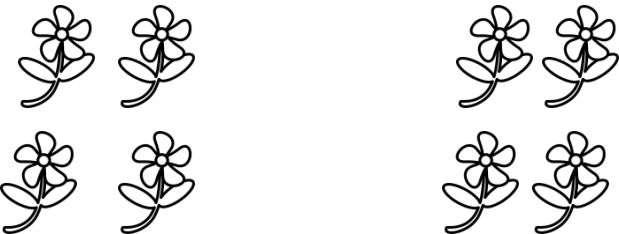
$$\begin{array}{r} 32 \\ \times 24 \\ \hline 8 \quad (4 \times 2) \\ 120 \quad (4 \times 30) \\ 40 \quad (20 \times 2) \\ 600 \quad (20 \times 30) \\ \hline 768 \end{array}$$

This moves to the more compact method.
Year 5/6

$$\begin{array}{r} 2 3 1 \\ 1342 \\ \times 18 \\ \hline 13420 \\ 10736 \\ \hline 24156 \\ 1 \end{array}$$

			$ \begin{array}{r} 74 \\ \times 63 \\ \hline 12 \\ 210 \\ 240 \\ + 4200 \\ \hline 4662 \end{array} $
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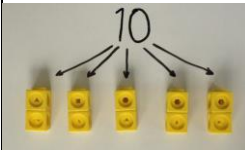
Division

Objective and Strategies	Concrete	Pictorial	Abstract
EYFS Solve problems including halving and sharing.		 There are eight sweets and four boys. How many sweets does each boy get?	Can you share the 8 sweets so that we have half each?
Year 1 Sharing objects into groups	  I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  $8 \div 2 = 4$	Share 9 buns between three people. $9 \div 3 = 3$ Year 2

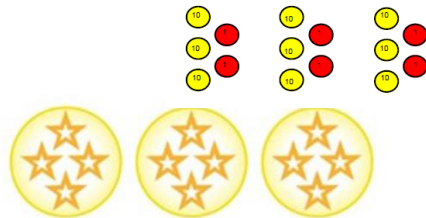
Year 1/2

Division as grouping

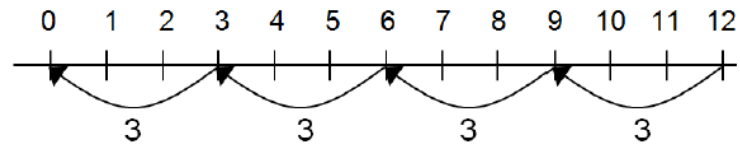
Divide quantities into equal groups.
Use cubes, counters, objects or place value counters to aid understanding.



$$96 \div 3 = 32$$



Use a number line to show jumps in groups. The number of jumps equals the number of groups.



Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.

Year 2



$$20 \div 5 = ?$$

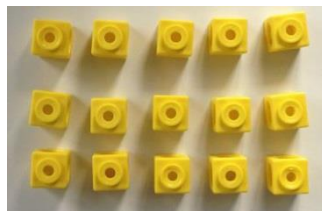
$$5 \times ? = 20$$

$$28 \div 7 = 4$$

Divide 28 into 7 groups.
How many are in each group?

Year 2

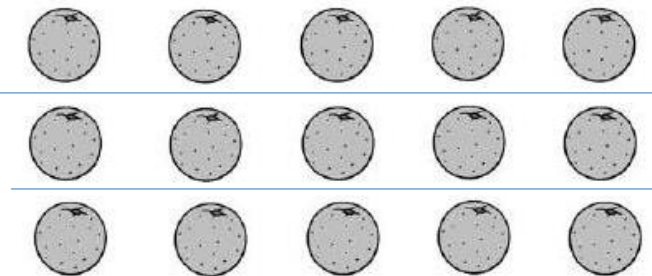
Division within arrays



Link division to multiplication by creating an array and thinking

about the number sentences that can be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$



Draw an array and use lines to split the array into groups to make multiplication and division sentences.

Find the inverse of multiplication and division sentences by creating four linking number sentences.

$$7 \times 4 = 28$$

$$4 \times 7 = 28$$

$$28 \div 7 = 4$$

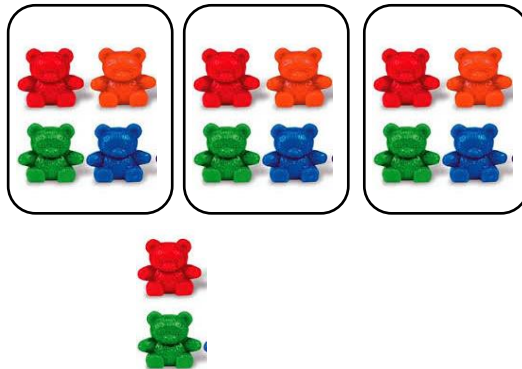
$$28 \div 4 = 7$$

Year 3

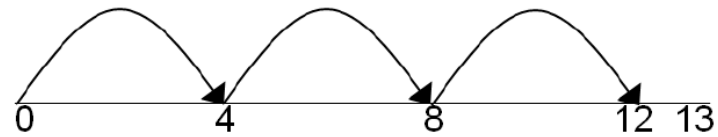
Division with a remainder

$$14 \div 3 =$$

Divide objects between groups and see how much is left over.



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.



Complete written divisions and show the remainder using r.

$$29 \div 8 = 3 \text{ REMAINDER } 5$$

↑ ↑ ↑ ↑
dividend divisor quotient remainder

Year 3 and 4

Short division

Year 5

Tens Units

3 2

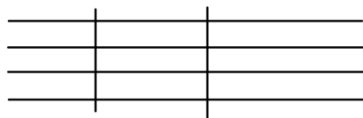
3



Use place value counters to divide using the bus stop method alongside



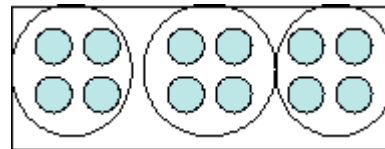
Calculations
 $42 \div 3$



$$42 \div 3 =$$

Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.

Pupils can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.



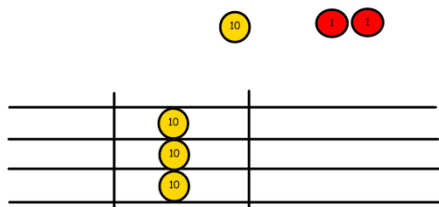
Encourage them to move towards counting in multiples to divide more efficiently.

Begin with divisions that divide equally with no remainder.

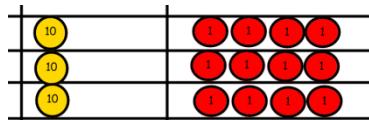
$$\begin{array}{r} 218 \\ 3 \overline{) 654} \end{array}$$

Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 258} \end{array}$$



We exchange this ten for ten ones and then share the ones equally among the groups.

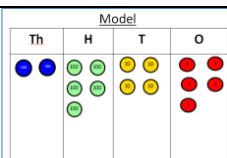


We look how much in 1 group so the answer is 14.

Finally move into decimal places to divide the total accurately.

$$\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \\ \underline{35} \\ 16 \\ \underline{15} \\ 1 \\ \underline{0} \\ 10 \\ \underline{0} \\ 0 \end{array}$$

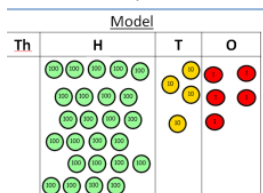
Year 6 Long division



$$2544 \div 12$$

How many groups of 12 thousands do we have? None

Exchange 2 thousand for 20 hundreds.



$$12 \overline{) 2544} \begin{array}{l} 0 \\ 2 \\ 0 \end{array}$$

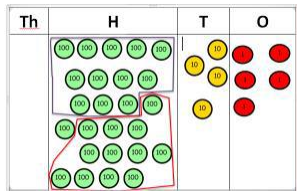
How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with

Instead of using physical counters, students can draw the counters and circle the groups on a whiteboard or in their books.

Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.

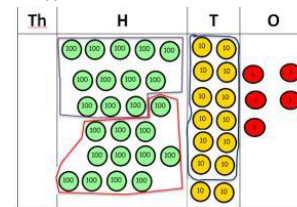
$$\begin{array}{r} 0318r5 \\ 20 \overline{) 6365} \\ \underline{60} \\ 36 \\ \underline{20} \\ 165 \\ \underline{160} \\ 5 \end{array}$$

one.



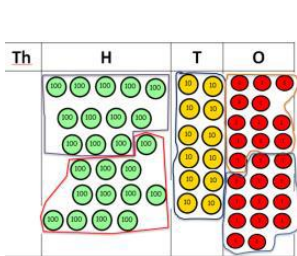
$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2



$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2



$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$